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## PHILMATH SEMINAR

### *Research seminar in the philosophy of mathematics and the philosophy of logic*

Organizers: Fabrice Pataut (SND - UMR 8011 Sciences, Normes, Démocratie, CNRS et Sorbonne Université) and Francesca Poggiolesi (IHPST - (UMR 8590, Institut d'Histoire et de Philosophie des Sciences et des Techniques, CNRS et Université Paris 1 Panthéon- Sorbonne)

### **Benedict Eastaugh (Université de Warwick) donnera une conférence sur le thème "Arrow's theorem in infinite societies: a reverse mathematical approach"**

**La conférence aura lieu le lundi le lundi 30 janvier 2023, de 17h00 à 19h00 à l'IHPST : 13, rue du Four, 75006 Paris, 2ème étage, Salle de conférences.**

Il est également possible de suivre la conférence et de participer à la discussion qui suivra par Zoom :

<https://pantheonsorbonne.zoom.us/j/9843978525?pwd=c09nQUxhUXpBMLlHNUoyR1ZsNXNZdz09>  
ID : 984 3978 5525 - Passcode : 373665

#### Résumé

Arrow's theorem in infinite societies: a reverse mathematical approach

Arrow's impossibility theorem appears to place substantial limits on the existence of methods for social decision-making that are fair, rational, and uniform. The theorem shows that there is no way of aggregating individual preferences into an overall social preference or general will, known as a social welfare function, assuming only that any such social welfare function must satisfy conditions of unanimity, independence, and non-dictatorship. Arrow's theorem has therefore been of major interest in economics and political philosophy as well as many other fields, including philosophy of science. It has also been the subject of many different formalisations, using modal logic, first-order logic, dependence logic, and intuitionistic logic. This talk presents a new approach to formalising Arrow's theorem and related results in social choice theory, with a focus on infinitary counterexamples to Arrow's theorem, since although the theorem holds for finite societies it fails in infinite ones. I will show that such infinitary counterexamples necessarily involve appeal to non-computable objects, namely non-principal ultrafilters, and that therefore it is difficult to regard them as able to act as ideal social planners or as genuinely expressing the general will of a society. However, these non-computable social welfare functions are also not overly complex (in terms of the computability-theoretic and proof-theoretic hierarchies used to measure this complexity), and can therefore be proved to exist in a system with the same proof-theoretic strength as first-order Peano arithmetic.